



Barnfields Primary School

Calculation Progression Policy

Subject Quest: Discover the properties of different relationships and to explain universal truths from a very basic set of commonly accepted belief system of numbers and symbols.

Rationale

At Barnfields Primary School, we believe in equipping all of our children to become fluent and proficient Mathematicians. The aim of this Calculation Progression Policy is to ensure all of our children are able to use a range of mental calculation strategies effectively, efficiently and flexibly. In partnership with this, it is imperative that children can also use a range of written methods accurately and appropriately. When children are able to employ both of these strategies effectively, they can then solve problems and reason mathematically efficiently. Underpinning all this, is the belief that all children can acquire a competent and confident understanding of numbers and the number system combined with an instant recall of a set of age-appropriate number facts. This knowledge and skillset combined with the automaticity of known facts will ensure our children are able to explore Maths in depth and with success using appropriate vocabulary and skills in order to do so effectively.

Concrete, Pictorial, Abstract approach

The Concrete, Pictorial, Abstract approach (CPA) is a highly effective approach to teaching that develops a deep and sustainable understanding of maths in pupils. It is an essential technique within the Singapore method of teaching maths for mastery which is now used to teach Maths in the majority of schools across the UK and employed at Barnfields Primary School. Children can find Maths difficult because it is abstract.

The CPA approach builds on children's existing knowledge by introducing abstract concepts in a concrete and tangible way. It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems. Concrete is the "doing" stage. During this stage, students use concrete objects to model problems. Unlike traditional maths teaching methods where teachers demonstrate how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects. With the CPA framework, every abstract concept is first introduced using physical, interactive concrete materials.


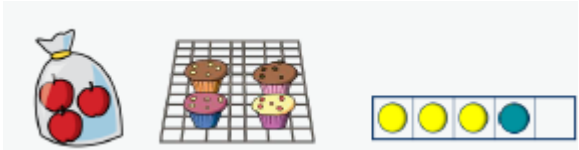

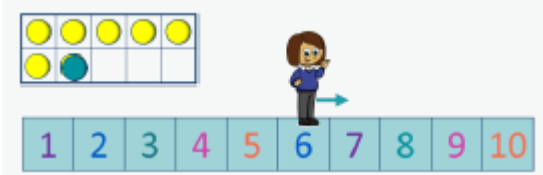

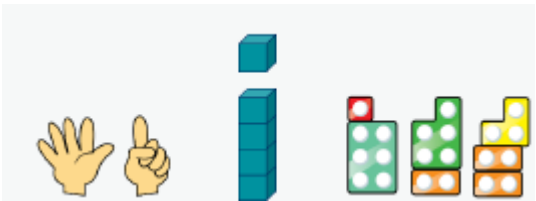
- Concrete is the "doing" stage. During this stage, children use concrete objects to model problems. Unlike traditional maths teaching methods where teachers demonstrate how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects.
- Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem. Building or drawing a model makes it easier for children to grasp difficult abstract concepts (for example, fractions). Simply put, it helps students visualise abstract problems and make them more accessible.


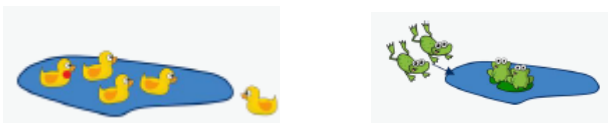
- Abstract is the “symbolic” stage, where children use abstract symbols to model problems. Students will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, −, x, ÷) to indicate addition, multiplication or division. It is important to note this is not a linear approach and teachers will move back and forth between the CPA stages as required.

Early Years

In Reception, calculation is taught through practical, hands-on experiences that develop children’s understanding of number. Pupils explore counting, addition and subtraction using concrete resources, play-based activities and real-life contexts. We focus on building strong number sense by encouraging children to represent quantities in different ways, use mathematical language and explain their thinking. Through guided exploration and structured play, children develop confidence in solving simple problems and begin to recognise patterns and relationships between numbers.

Addition in Reception


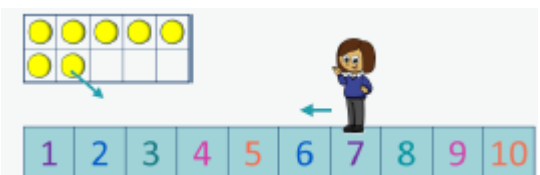
Concept	Concrete	Pictorial	Abstract
Conceptually subitising to 5	 <p>What do you see? How do you see it?</p>		
1 more	<p>1 more than... is...</p> 		
Notice the composition of numbers within 10	 <p>How many...? How many altogether?</p>	<p>How many ways can you make?</p> 	




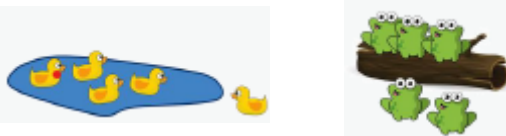

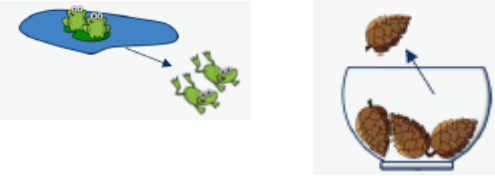

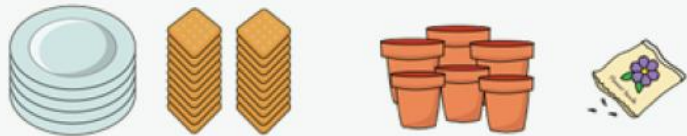
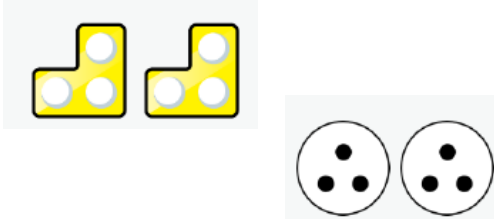
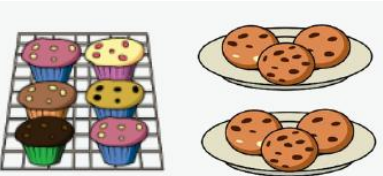
Combine 2 groups	<p>There are... There are... There are...altogether.</p> 	<p>...and...make...</p> 	
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Subtraction in Reception



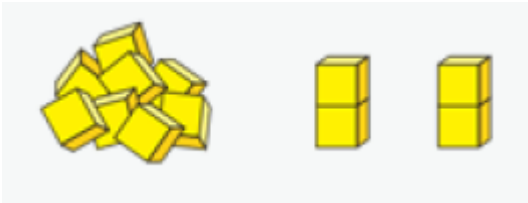

Concept	Concrete	Pictorial	Abstract
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Conceptually subitise to 5	<p>What do you see? How do you see it?</p> 		
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1 less		<p>1 less than...is...</p> 	
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<p>Notice the composition of numbers within 10</p>	 <p>How many...?</p> <p>How many altogether?</p>		
<p>Partition a number into two or parts</p>	<p>There are...altogether. I can see...here and...there.</p> 	 <p>...and...make...</p>	
<p>Take away (reducing a quantity)</p>	<p>First...Then...Now</p> 	 <p>I have.. I take...away... Now I have...</p>	
<p>Multiplication in Reception</p>			
<p>Concept</p>	<p>Concrete</p>	<p>Pictorial</p>	<p>Abstract</p>
<p>Making equal groups</p>		<p>There are ... groups of ... There are ... altogether.</p> 	
<p>Doubling to 10</p>		<p>Double ... is is double ...</p> 	

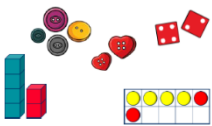
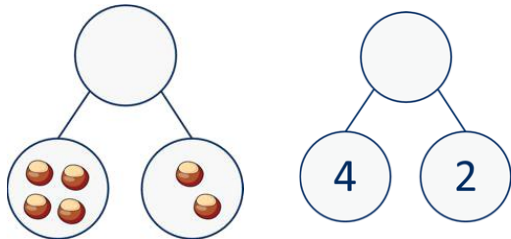
Division in Reception

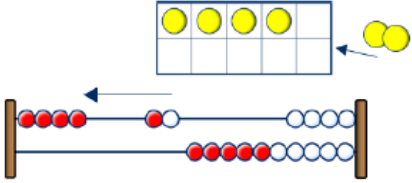
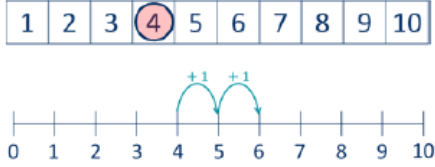



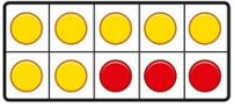
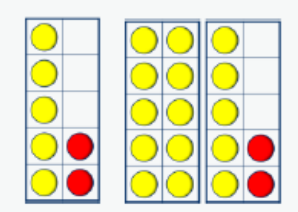
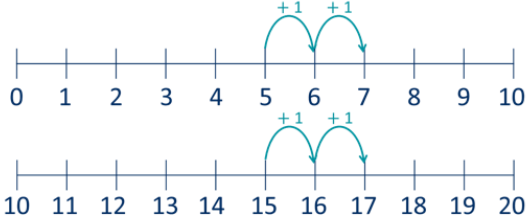
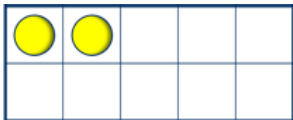
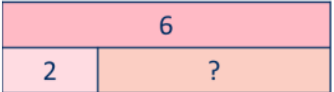
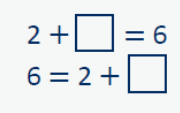
Division in Reception			
Concept	Concrete	Pictorial	Abstract
Sharing		<p>There are ... altogether. They are shared equally between ... groups.</p> 	
Grouping		<p>There are ... groups of ... There are ... altogether.</p> 	

Key Stage 1

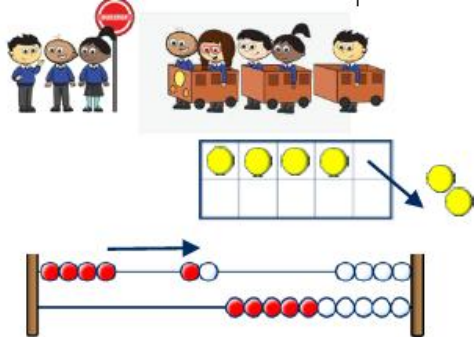
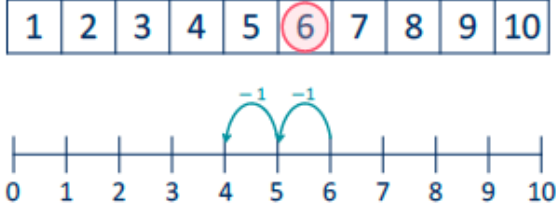
Children develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on and counting back, but they should learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. They learn how to use an understanding of 10s and 1s to develop their calculation strategies, especially in addition and subtraction.

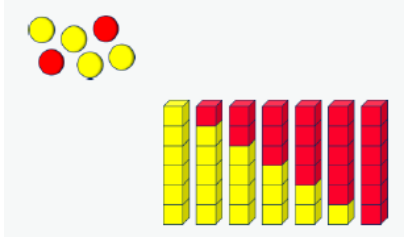
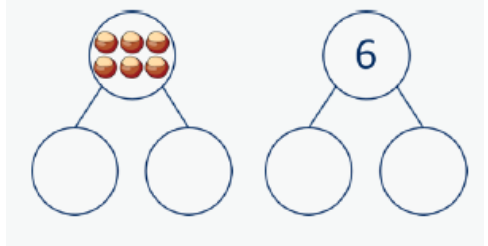
Year 1 addition


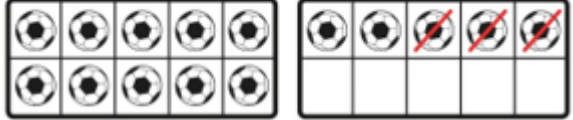
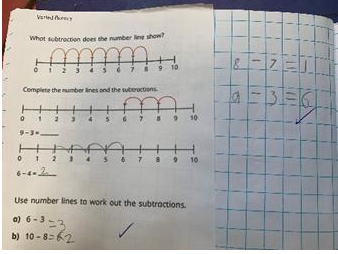
Year 1 addition			
Concept	Concrete	Pictorial	Abstract
Add together (aggregation)	<p>There are ... There are ... There are ... altogether.</p> 		<p>... plus ... is equal to is equal to + ... $4 + 2 = 6$ $2 + 4 = 6$ $6 = 4 + 2$</p>

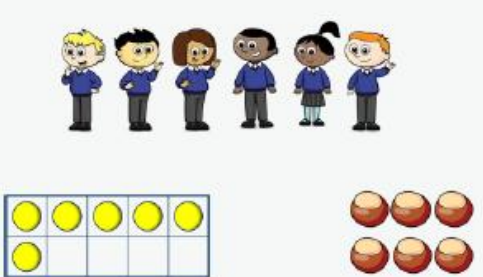
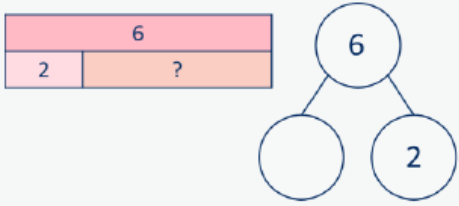

<p>Add more augmentation</p>	<p>First...then...now...</p> 	<p>I start at ... I jump on ... I land on ...</p> 	<p>plus ... is equal to ... is equal to ... + ...</p> $4 + 2 = 6$ $2 + 4 = 6$ $6 = 4 + 2$
<p>Knowing and finding number bonds within 10</p>	 $3 + 4 = 7$  $6 = 2 + 4$	<p>Use five and ten frames to represent key number bonds.</p>  $5 = 4 + 1$  $10 = 7 + 3$	<p>... plus ... is equal to ...</p> $6 + 0 = 6$ $5 + 1 = 6$ $4 + 2 = 6$ $3 + 3 = 6$ $2 + 4 = 6$ $1 + 5 = 6$ $0 + 6 = 6$
<p>Related facts within 20</p>	<p>I know that ... and ... = ... so ... and ... = ...</p> 		<p>What patterns do you notice?</p> $5 + 2 = 7$ $15 + 2 = 17$ $7 = 5 + 2$
<p>Missing numbers</p>		<p>If ... is the whole and ... is a part, the other part must be...</p> 	

Year 1 Subtraction

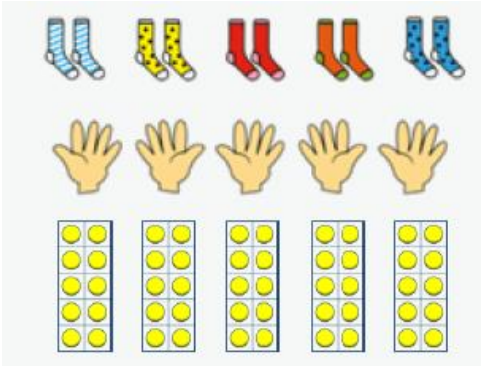

Counting back and taking away	First... Then... Now... 	I start at ... I jump back ... I land on ... 	... minus ... is equal to is equal to ... - ... $6 - 2 = 4$ $6 - 4 = 2$ $4 = 6 - 2$ $2 = 6 - 4$
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Bonds within 10 Focus on subtraction facts. Encourage children to notice patterns.	... is made of ... and and ... make ... 	... can be partitioned into ... and ... 	... minus ... is equal to ... $6 - 0 = 6$ $6 - 1 = 5$ $6 - 2 = 4$ $6 - 3 = 3$ $6 - 4 = 2$ $6 - 5 = 1$ $6 - 6 = 0$
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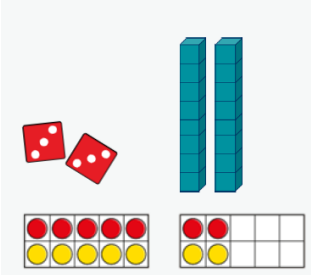
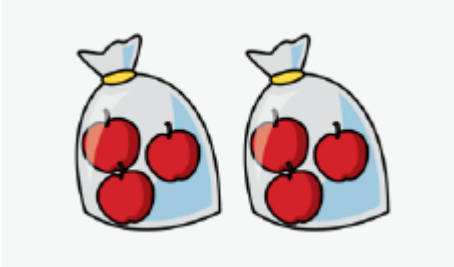
Subtraction within 20	Use a bead string to subtract 1s efficiently  $5 - 3 = 2$ $15 - 3 = 12$	 $5 - 3 = 2$ $15 - 3 = 12$	
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<p>Missing numbers Make links to known facts.</p>	<p>How many do you need to subtract to make ...?</p> 	<p>If ... is the whole and ... is a part, the other part must be...</p> 	<p>... minus ... is equal to ...</p> $6 - \square = 2$ $2 = 6 - \square$ 
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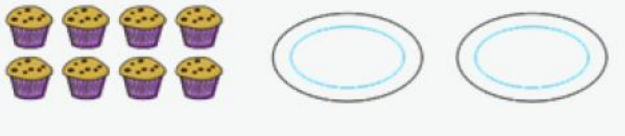


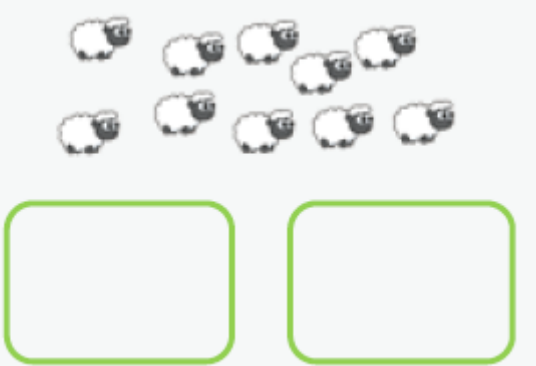
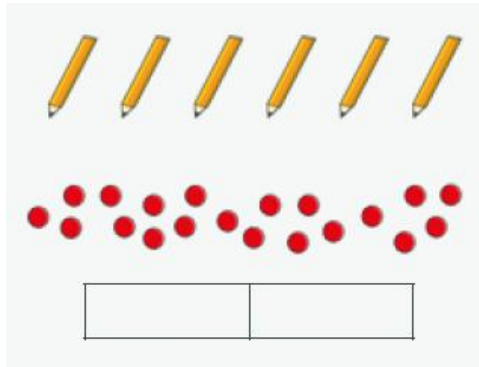
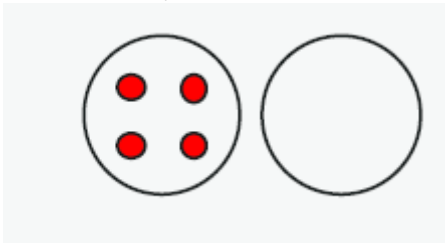
Year 1 multiplication

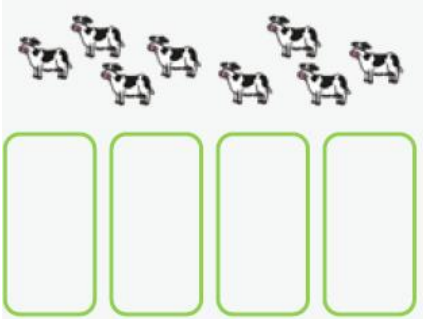
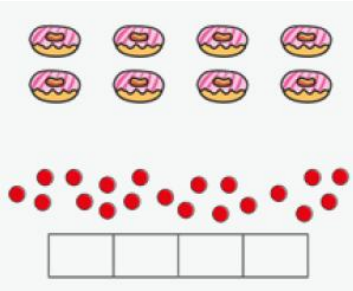
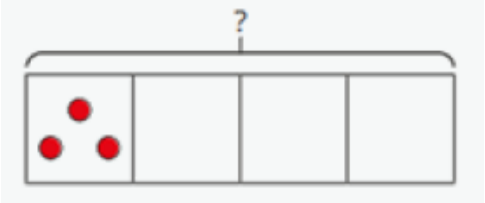
Concept	Concrete	Pictorial	Abstract																																																										
<p>Finding the total of equal groups by counting in 2s, 5s and 10s</p>	<p>Provide objects to count</p> 	<p>Continue to colour in ...s What do you notice?</p> <table border="1" data-bbox="990 625 1473 865"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> </table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	<p>Complete the number track/number line by counting in ...s.</p> <table border="1" data-bbox="1617 635 2065 689"> <tr><td>5</td><td>10</td><td>15</td><td>20</td><td></td><td></td><td></td><td></td></tr> </table> 	5	10	15	20				
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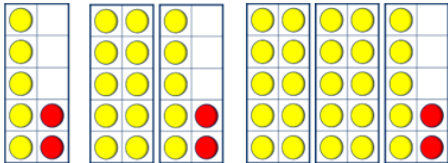
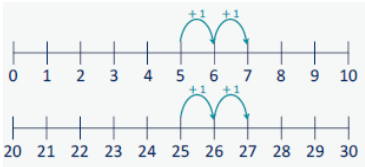
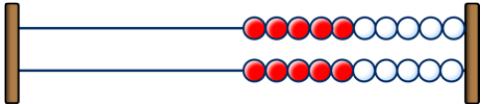
<p>Make arrays</p>	<p>There are ... rows of ... There are ... altogether. There are ... columns of ... There are ... altogether.</p> 		
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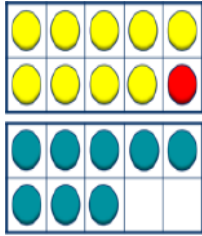
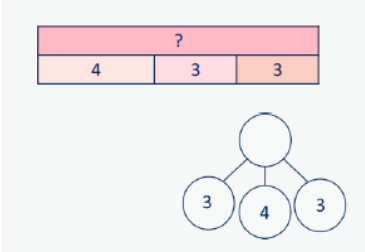

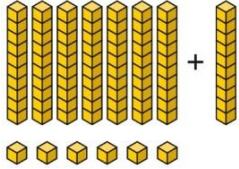
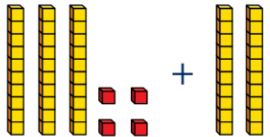
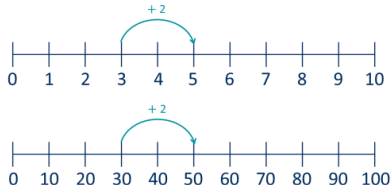
<p>Make doubles</p>			<p>Double ... is ... $2 + 2 = 4$</p>
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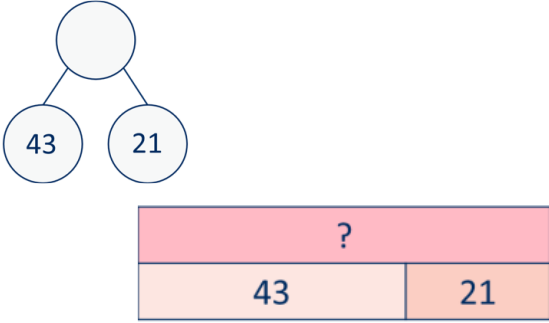
Year 1 Division

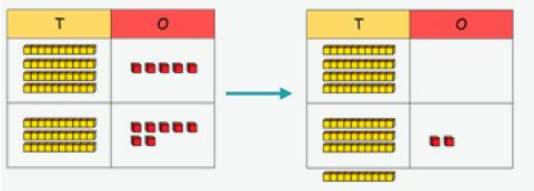
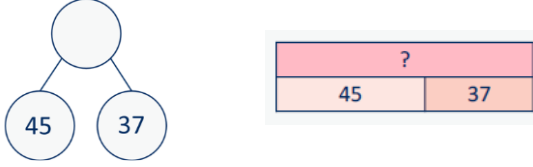
Concept	Concrete	Pictorial	Abstract
<p>Sharing</p>	<p>Share a set of objects into equal parts and work out how many are in each part.</p>  <p>There are ... in each group.</p>	<p>Sketch or draw to represent sharing into equal parts. This may be related to fractions.</p> 	<p>Take ... cubes. Share them between ...</p>  <p>12 shared between ... is ...</p>
<p>Find a half</p>	<p>To find half, I need to share into 2 equal groups.</p> 	<p>Half of ... is ...</p> 	<p>If ... is half, what is the whole?</p>  <p>4 is half of ...</p>

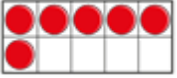
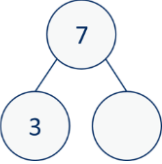
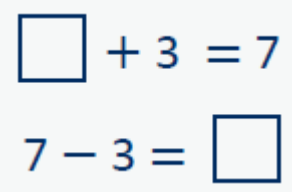
	There are ... in each group.		
Find a quarter	<p>To find a quarter, I need to share into 4 equal groups.</p> <p>There are ... in each group.</p> 	<p>A quarter of ... is ...</p> 	<p>If ... is one quarter, what is the whole?</p> <p>3 is one quarter of ...</p> 

Year 2 Addition			
Concept	Concrete	Pictorial	Abstract
<p>Add ones to any number (related facts)</p> <p>Make links to known facts.</p>	<p>I know that ... and ... = ...</p> <p>so ... and ... = ...</p> 	<p>... more than ... is ...</p> <p>so ... more than ... is ...</p> 	<p>What do you notice?</p> <p>Can you continue the pattern?</p> <p>$5 + 2 = 7$</p> <p>$15 + 2 = 17$</p> <p>$25 + 2 = 27...$</p>
<p>Add three 1-digit numbers</p> <p>Prompt children to understand that addition can be done in any order</p>	<p>... and ... are a bond to 10</p> <p>$10 + ... = ...$</p> 	<p>Double ... + ... = ...</p>	<p>What do you notice?</p> <p>Which addition is the easiest to calculate?</p> <p>$8 + 9 + 1 =$</p> <p>$8 + 1 + 9 =$</p> <p>$9 + 1 + 8 =$</p>

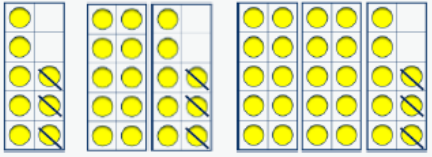
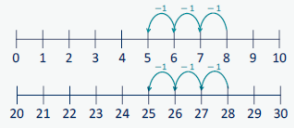
<p>and to make links to known facts.</p>																																																																																																							
<p>Adding a multiple of 10 to a 2 digit number</p>	<p>Add the 10s and then recombine.</p>  <p><i>27 is 2 tens and 7 ones. 50 is 5 tens.</i></p> <p><i>There are 7 tens in total and 7 ones. So, 27 + 50 is 7 tens and 7 ones.</i></p>	<p>Add the 10s and then recombine.</p>  <p><i>66 is 6 tens and 6 ones. 66 + 10 = 76</i></p> <p>A 100 square can support this understanding.</p> <table border="1" data-bbox="1160 788 1397 1027"> <tbody> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> <tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr> <tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr> <tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr> </tbody> </table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	<p>Add the 10s and then recombine.</p> <p>$37 + 20 =$ $30 + 20 = 50$ $50 + 7 = 57$ $37 + 20 = 57$</p>
1	2	3	4	5	6	7	8	9	10																																																																																														
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<p>Adding a multiple of 10 to a 2-digit number</p>	<p>... tens + ... tens = ... tens ... tens and ... ones = ...</p> 		<p>Make links to known facts within ten.</p> <p>$3 + 2 = 5$ $30 + 20 = 50$</p>																																																																																																				

<p>Add 2-digit numbers (not across a ten) Lining up ones and tens in columns will support with later written methods.</p>			<p>3 ones + 1 one = 4 ones 4 tens + 2 tens = 6 tens 6 tens + 4 ones = 64</p>
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<p>Add 2-digit numbers (across a ten) Begin to exchange 10 ones for 1 ten.</p>			<p>5 ones + 7 ones = 12 ones 12 ones = 1 ten and 2 ones 4 tens + 3 tens + 1 ten = 8 tens 8 tens and 2 ones = 82</p>
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<p>Missing numbers Solve missing number problems and use the inverse to check.</p>	<p>How many more do you need to make ...?</p>  $6 + \square = 10$ $10 - \square = 6$	<p>If ... is a whole and ... is a part, then ... is the other part.</p> 	<p>... can be partitioned into ... and ...</p> 
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Year 2 Subtraction			
Concept	Concrete	Pictorial	Abstract

<p>Subtract ones from any number</p>	<p>I know that ... minus ... = ... so ... minus ... = ...</p> 	<p>... less than ... is ... so ... less than ... is ...</p> 	<p>What do you notice? Can you continue the pattern? 8 - 3 = 5 18 - 3 = 15 28 - 3 = 2 ...</p>
--------------------------------------	---	---	---

Subtracting a single digit number bridging 10

Bridge 10 by using known bonds.

$35 - 6$
I took away 5 counters, then 1 more.

Bridge 10 by using known bonds.

Make links with related facts.

Subtracting a single digit number using exchange

$25 - 7 =$

Subtracting a 2 digit number (no exchange)

Subtract the ones then the tens

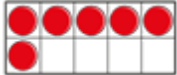
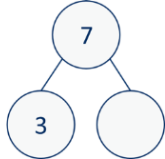
$38 - 16 = 22$

$38 - 16 = 22$



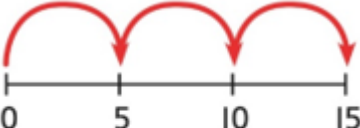

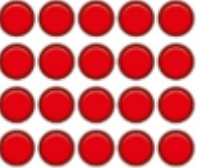
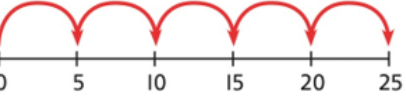
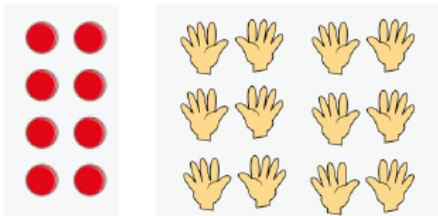
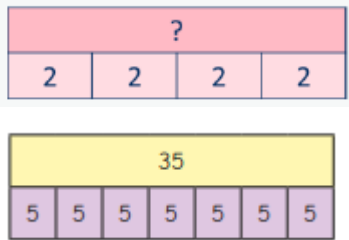
Subtracting a 2 digit number with an exchange

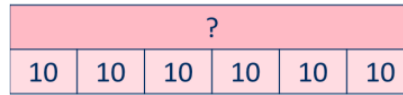
I need to make an exchange because I do not have enough ones to subtract ... ones.

$43 - 25 =$

<p>Missing numbers Solve missing number problems and use the inverse to check.</p>	<p>How many do you need to subtract to make ...?</p>  $6 + \square = 10$ $10 - \square = 6$	<p>If ... is a whole and ... is a part, then ... is the other part.</p> 	<p>... can be partitioned into ... and ...</p> $\square + 3 = 7$ $7 - 3 = \square$
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Year 2 Multiplication

Concept	Concrete	Pictorial	Abstract
<p>Equal groups and repeated addition</p>		 <p><i>3 groups of 5 15 in total</i></p>	 $5 + 5 + 5 = 15$ $3 \times 5 = 15$
<p>Using arrays to represent multiplication and support understanding</p>		<p>Four groups of 5</p> 	 $5 \times 5 = 25$
<p>Learning 2x, 5x and 10 x tables facts</p>			$1 \times 10 = 10 \quad 10 = 1 \times 10$ $2 \times 10 = 20 \quad 20 = 2 \times 10$ $3 \times 10 = 30 \quad 30 = 3 \times 10$

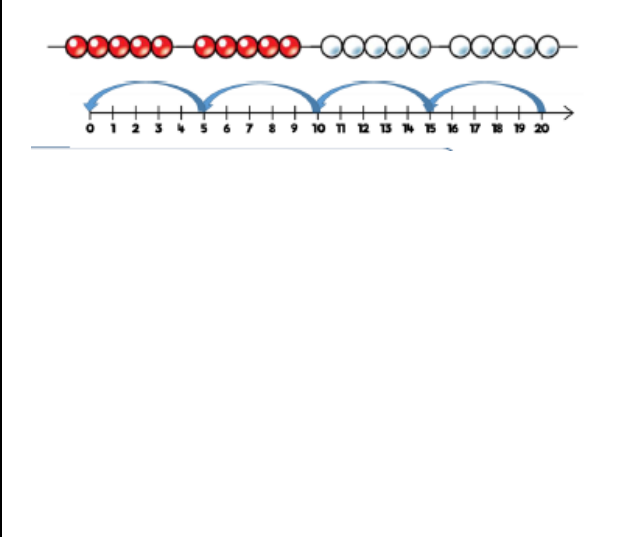


Year 2 Division

Concept Concrete Pictorial Abstract

Grouping
equally

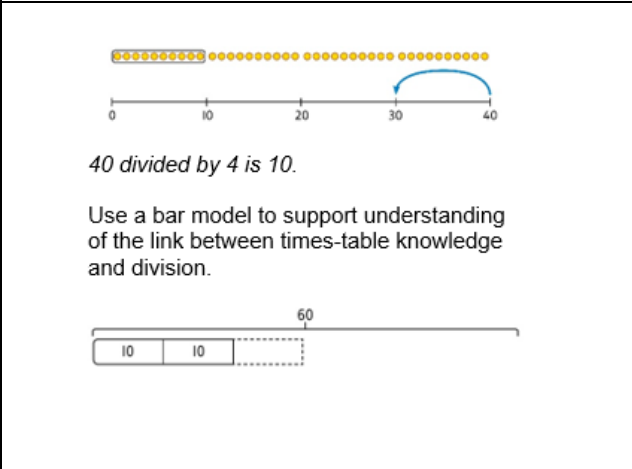
There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?



Abstract
 $20 \div 5 = 4$

Using known
times tables
to solve
divisions

4 groups of 5 cars is 20 cars in total.
20 divided by 4 is 5.

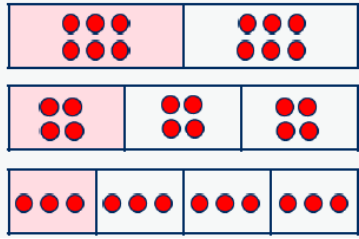

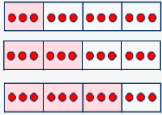
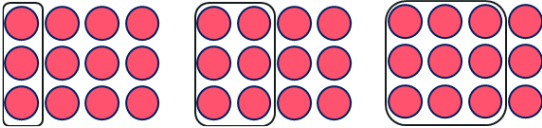


$1 \times 10 = 10$
 $2 \times 10 = 20$
 $3 \times 10 = 30$
 $4 \times 10 = 40$
 $5 \times 10 = 50$
 $6 \times 10 = 60$
 $7 \times 10 = 70$
 $8 \times 10 = 80$

I used the 10 times-table to help me.
 $3 \times 10 = 30$.

I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3.

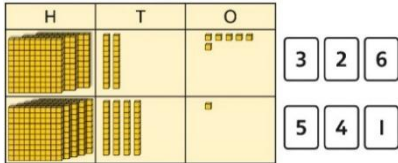
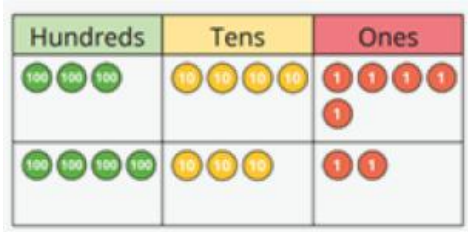
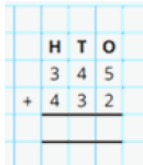
$3 \times 10 = 30$ so $30 \div 10 = 3$

<p>Unit Fractions $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{3}$</p>	<p>The objects have been shared fairly into ... groups.</p> 		<p>There are ... equal parts. There is ... part circled. $\frac{1}{\square}$ is circled.</p>
<p>Non- unit fractions Recognise $\frac{2}{4}$ and $\frac{3}{4}$</p>	<p>The objects have been shared fairly into ... groups.</p> 		<p>There are ... equal parts. There are ... parts circled. $\frac{\square}{\square}$ is circled.</p>

LOWER KEY STAGE 2

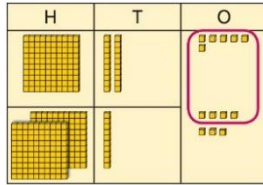
In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.

Year 3 Addition

Concept	Concrete	Pictorial	Abstract
<p>Add numbers with up to three digits using the formal written method of column addition- no exchange</p>	<p>Use place value equipment to make a representation of a calculation. This may or may not be structured in a place value grid.</p> <p><i>326 + 541 is represented as:</i></p> 	<p>Represent the place value grid with equipment to model the stages of column addition.</p> 	<p>Use a column method to solve efficiently, using known bonds. Children must understand how this relates to place value at every stage of the calculation.</p> 
<p>Add numbers with up to three digits using the</p>	<p>Use place value equipment to enact the exchange required.</p>	<p>Model the stages of column addition using place value equipment on a place value grid.</p>	<p>Use column addition, ensuring understanding of place value at every stage of the calculation.</p>

formal written method of column addition- with exchange

There are 13 ones.
I will exchange 10 ones for 1 ten.



H	T	O
3	6	7
+ 1	6	4
5	3	1

Note: Children should also study examples where exchange is required in more than one column, for example $185 + 318 = ?$

Complements to 100
Pairs of numbers which total 100

... plus ... is equal to 100

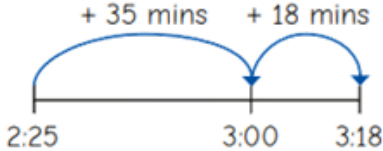
I add ... to get to the next 10, then ... to get to 100...

$38 + 62 = 100$
 $62 + 38 = 100$
 $100 = 38 + 62$
 $100 = 62 + 38$

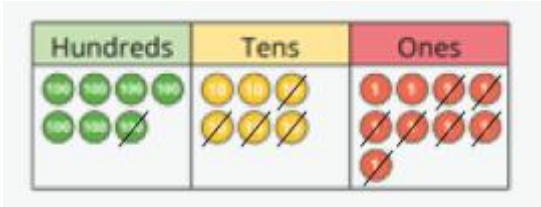
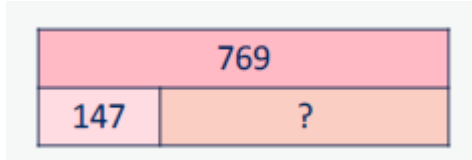
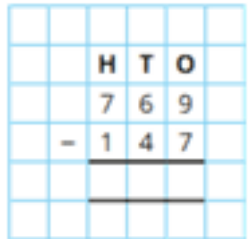
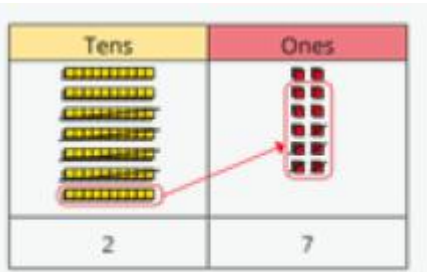

Add fractions with the same denominator within 1 whole
Make links with known facts.

When adding fractions with the same denominator, I only add the numerator.
... fifths + ... fifths = ... fifths

$\frac{1}{5} + \frac{1}{5}$
 $\frac{1}{5} + \frac{2}{5}$
 $\frac{1}{5} + \frac{3}{5}$

<p>Calculate the duration of events Find durations of time between a given start and end point. Children will need to calculate complements to 60</p>			<p>From ... to ... o'clock is ... minutes. From ... o'clock to ... is ... minutes. The total time taken is ... minutes.</p> 
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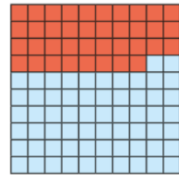
Year 3 Subtraction

Concept	Concrete	Pictorial	Abstract
<p>Subtract numbers with up to three digits using formal written methods of column subtraction without exchange.</p>	<p>... ones – ... ones = ... ones ... tens – ... tens = ... tens ... hundreds – ... hundreds = ... hundreds</p> 		
<p>Subtract numbers with up to three digits using formal written methods of column subtraction with exchange.</p>	<p>I need to subtract ... ones. I do/do not need to make an exchange. I need to subtract ... tens. I do/do not need to make an exchange. I can exchange 1 ... for 10 ...</p>		

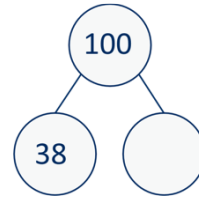


Complements to 100
Focus on subtraction facts.
Encourage children to notice patterns.

100 minus ... is equal to ...



I subtract ... tens, then I subtract ... ones.



100	
38	?

$$100 - 38 = 62$$

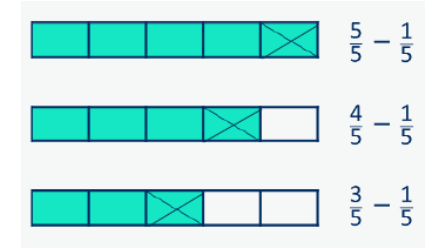
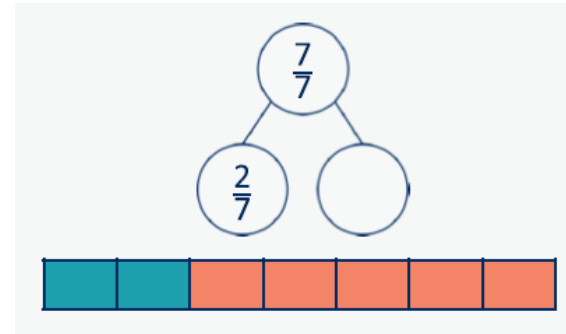
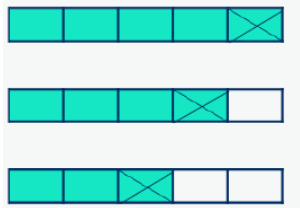
$$100 - 62 = 38$$

$$62 = 100 - 38$$

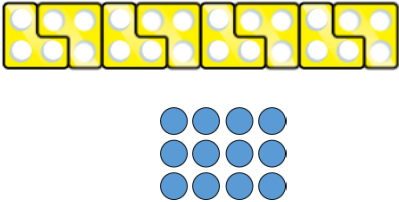
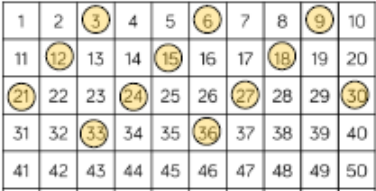

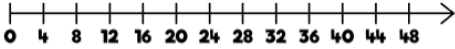
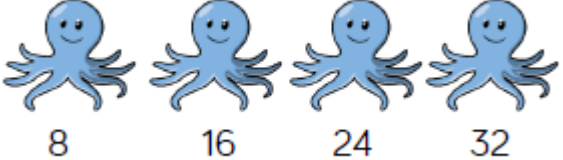
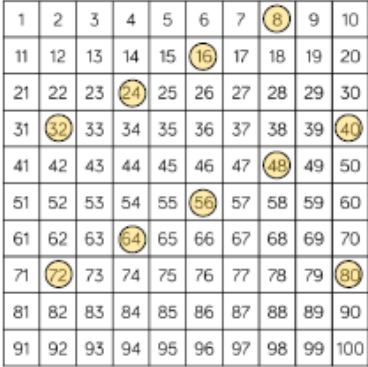
$$38 = 100 - 62$$

Subtract fractions with the same denominator within 1 whole
Make links with known facts.

When subtracting fractions with the same denominator, I only subtract the numerator.
... fifths - ... fifths = ... fifths



Year 3 Multiplication

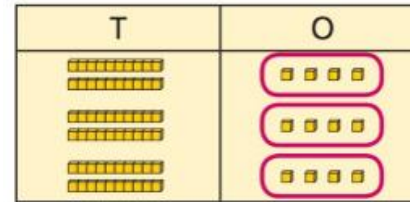
Concept	Concrete	Pictorial	Abstract
Recall and use multiplication and division facts for the 3x tables			$1 \times 3 = 3$ $2 \times 3 = 6$ $3 \times 3 = 9$
Recall and use multiplication and division facts for the 4x tables			$3 \times 4 = 12$ $4 \times 4 = 16$ $5 \times 4 = 20$
Recall and use multiplication and division facts for the 8x tables			$5 \times 8 = 40$ $6 \times 8 = 48$ $7 \times 8 = 56$

Multiply 2 digit numbers by a one digit number

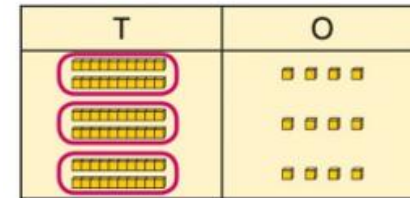
... tens multiplied by ... is equal to ... tens.
 ... ones multiplied by ... is equal to ... ones.



$$3 \times 24 = ?$$



$$3 \times 4 = 12$$



$$3 \times 20 = 60$$

$$60 + 12 = 72$$

$$3 \times 24 = 72$$

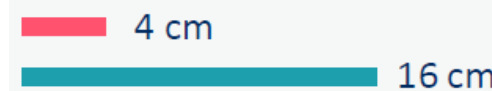
	H	T	O	
		3	4	
x			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

Scaling
 Children focus on multiplication as scaling (... times the size) as opposed to repeated addition.





























There are 3 times as many triangles as circles.



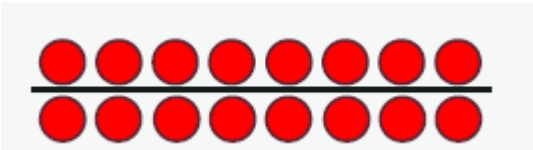
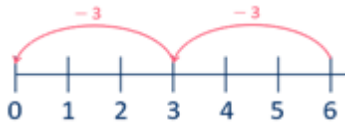








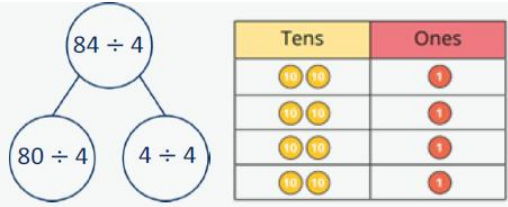




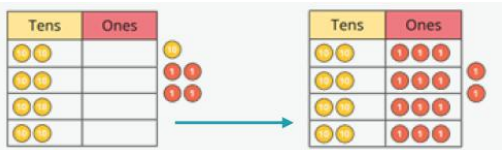
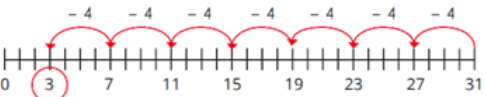
... is ... times the size of ...
 ... is ... times the length/height of ...

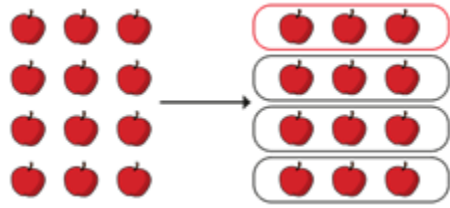
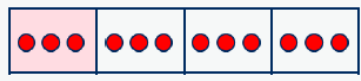


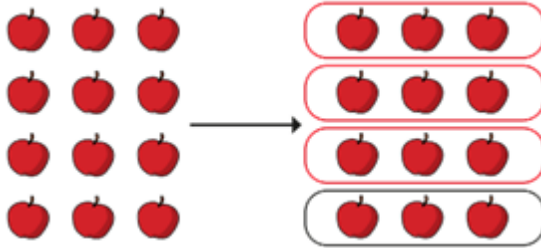

Miss Smith is twice the height of Jo.
 If Mrs Smith is 180cm, how tall is Jo?

<p>Correspondence problems (How many ways?) Encourage children to work systematically to find all the different possible combinations.</p>	<p>For every ... , there are ... possible ... There are ... × ... possibilities altogether.</p> 	<table border="1" data-bbox="1205 108 1512 481"> <thead> <tr> <th>hats</th> <th>scarves</th> </tr> </thead> <tbody> <tr> <td>blue </td> <td> </td> </tr> <tr> <td>orange </td> <td> </td> </tr> <tr> <td>purple </td> <td> </td> </tr> </tbody> </table>	hats	scarves	blue 	 	orange 	 	purple 	 	<p>For every hat, there are two possible scarves. $3 \times 2 = 6$ There are 6 possibilities altogether.</p>
hats	scarves										
blue 	 										
orange 	 										
purple 	 										

Year 3 Division

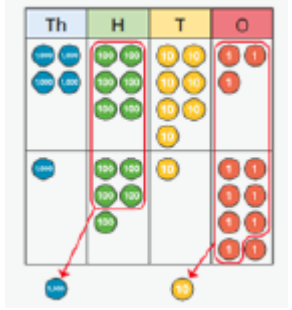
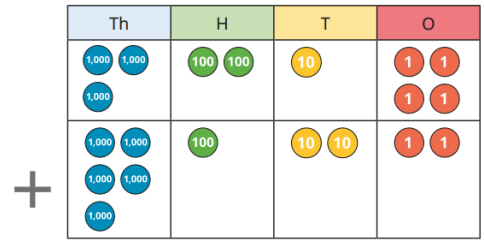
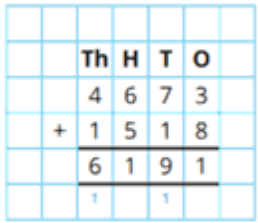
Concept	Concrete	Pictorial	Abstract						
<p>Divide by 3, 4 and 8 Use related multiplication facts</p>			<p>$2 \times 4 = 8$ $8 \div 4 = 2$</p>						
<p>Divide a 2 digit number by a 1 digit number without an exchange</p>	<table border="1" data-bbox="481 805 813 1042"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> </tbody> </table>	Tens	Ones						<p>$60 \div 2 = 30$ $4 \div 2 = 2$ $64 \div 2 = 32$</p>
Tens	Ones								
									
									
<p>Divide a 2 digit number by 1 digit number with a remainder</p>		 <p style="text-align: center;">$64 \div 2 = 32$</p>	<p>There are ... groups of ... There are ... remaining.</p> <p>$29 \div 2 = 14$ remainder 1</p>						


Unit fractions of a set of objects	<p>The whole is divided into ... equal parts.</p>  <p>$\frac{1}{4}$ of 12 apples is 3 apples</p>	<p>$\frac{1}{4}$ of 12 is 3</p> 	1/3 of 36 is 12
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Non-unit fractions of a set of objects	<p>The whole is divided into ... equal parts.</p>  <p>$\frac{3}{4}$ of 12 apples is 9 apples.</p>	<p>$\frac{2}{3}$ of 36 is 24</p> 	$\frac{3}{4}$ of 12 is 9.
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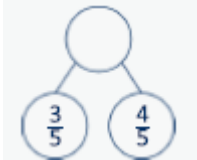
Year 4 Addition

Concept	Concrete	Pictorial	Abstract
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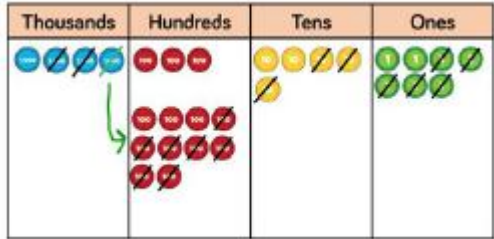
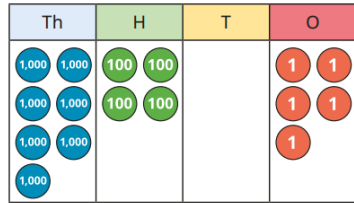
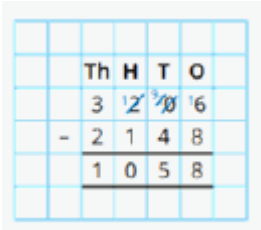
Add up to two 4 digit numbers up to 3 exchanges	<p>There are ... ones/tens/hundreds so I do/do not need to make an exchange.</p> 	<p>Calculate $3,214 + 5,122$ Use the place value chart to help you.</p> 	
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<p>Add 1s, 10s and 100s to a 4-digit number</p>	<p>The ones/tens/hundreds/thousands column will increase by ...</p>		$3,425 + 3 =$ $3,425 + 30 =$ $3,425 + 300 =$ $3,425 + 3,000 =$
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<p>Add decimal numbers in the context of money Emphasis on partitioning and use of number lines rather than formal written calculations.</p>	<p>... pence + ... pence = ... pence ... pounds + ... pounds = ... pounds</p> 		$45p + 25p = 70p$ $£2 + £3 = £5$ $£5 + 70p = £5.70$
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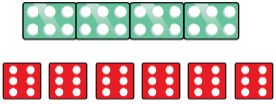
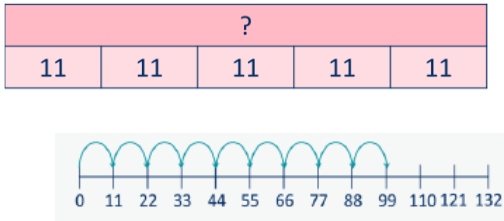
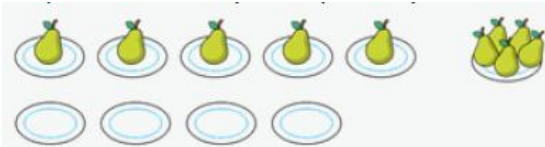
<p>Add fractions and mixed numbers with the same denominator beyond 1 whole</p>	<p>When adding fractions with the same denominator, I only add the numerator. ... fifths + ... fifths = ... fifths</p> 		$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$
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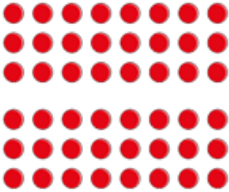
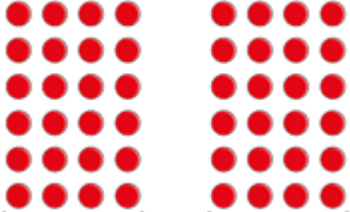
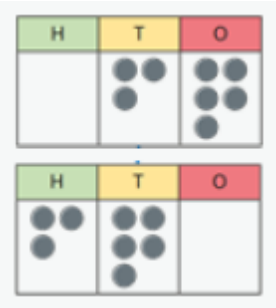
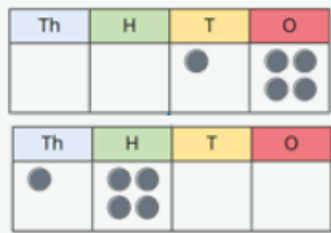
Year 4 Subtraction

Concept	Concrete	Pictorial	Abstract
<p>Subtract two 4 digit numbers- up to 3 exchanges.</p>		<p>c) 7,405 - 404</p> 	

Subtract 1s, 10s, 100s and 1,000s from a 4-digit number	The ones/tens/hundreds/thousands column will decrease by ...		What patterns do you notice? $4,356 - 3 =$ $4,356 - 30 =$ $4,356 - 300 =$ $4,356 - 3,000 =$
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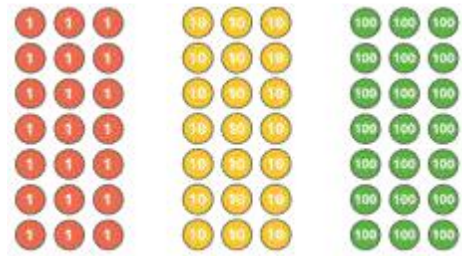
Year 4 Multiplication

Concept	Concrete	Pictorial	Abstract
Recall and apply multiplication facts up to 12×12	<p style="text-align: center;">... groups of ... = ... times ... is equal to \times ... =</p> 		$3 \times 6 = 18$ $6 \times 3 = 18$
Multiply by 1 and 0	<p style="text-align: center;">Any number multiplied by 1 is equal to ... Any number multiplied by 0 is equal to ...</p> 		<p style="text-align: center;">... \times ... = ... Multiplication</p> $1 \times 1 = 1$ $2 \times 1 = 2$ $3 \times 1 = 3$ $4 \times 1 = 4$ $1 \times 0 = 0$ $2 \times 0 = 0$ $3 \times 0 = 0$ $4 \times 0 = 0$

<p>Multiply 3 numbers Children use their understanding of commutativity to multiply more efficiently.</p>	<p>To work out ... × ... × ..., I can first calculate ... × ... and then multiply the answer by ...</p>		$4 \times 2 \times 3 = 8 \times 3 = 24$ $2 \times 3 \times 4 = 6 \times 4 = 24$ $3 \times 4 \times 2 = 12 \times 2 = 24$
<p>Factor pairs</p>	<p>$12 = \dots \times \dots$, so $\dots \times 12 = \dots \times \dots \times \dots$</p> 		$6 \times 8 = 6 \times 4 \times 2$ $6 \times 8 = 24 \times 2$
<p>Multiply by 10 and 100</p>	<p>When I multiply by 10, the digits move ... place value column to the left. ... is 10 times the size of ...</p> 	<p>When I multiply by 100, the digits move ... place value columns to the left. ... is 100 times the size of ...</p> 	$35 \times 10 = 350$ $14 \times 100 = 1,400$

Related facts
Use knowledge of multiplying by 10 and 100 to scale times-table facts.

... × ... ones is equal to ... ones
so ... × ... tens is equal to ... tens
and ... × ... hundreds is equal to ... hundreds.



$$3 \times 7 = 21$$

$$3 \times 70 = 210$$

$$3 \times 700 = 2,100$$

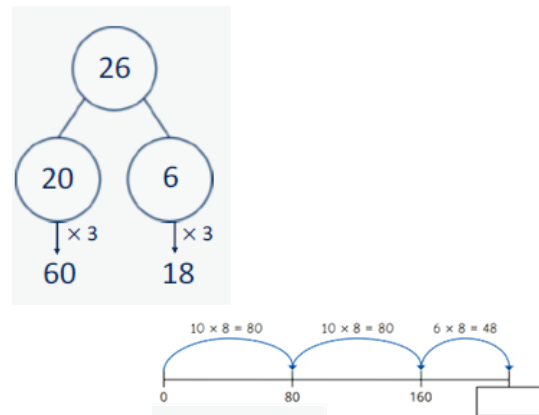
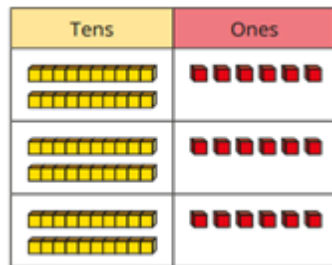
$$7 \times 3 = 21$$

$$7 \times 30 = 210$$

$$7 \times 300 = 2,100$$

Mental strategies
Partition 2 or 3-digit numbers to multiply using informal methods.

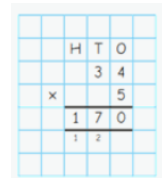
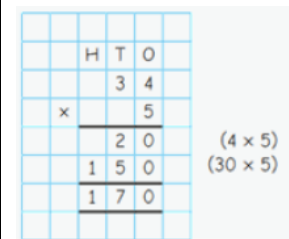
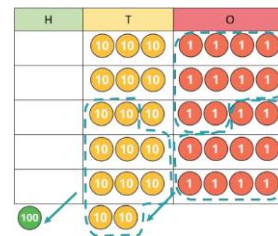
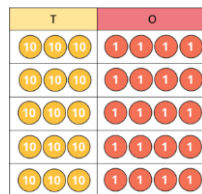
... tens multiplied by ... is equal to ... tens.
...ones multiplied by ... is equal to ... ones.
 $3 \times 26 = 60 + 18 = 78$



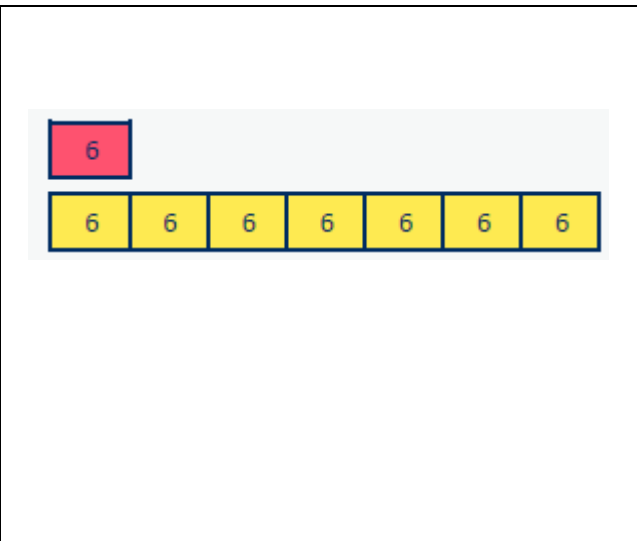
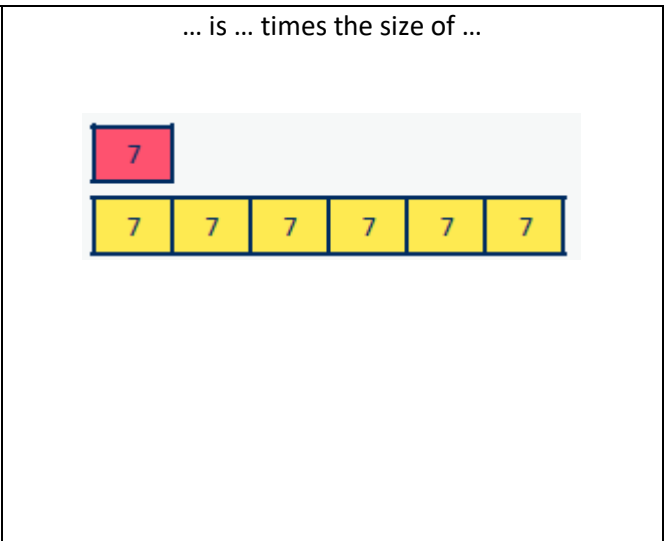
$$26 \times 8 = 80 + 80 + 48 = 208$$

Multiply a 2 or 3-digit number by a 1-digit number
The short multiplication method is introduced for the first time, initially in an expanded form.

To multiply a 2-digit number by ... , I multiply the ones by ... and the tens by ...
To multiply a 3-digit number by ... , I multiply the ones by ... , the tens by ... and the hundreds by ...



Scaling
Children focus on multiplication as scaling (... times the size).



A computer mouse costs £7 A red ribbon is 6 cm.
A keyboard costs 6 times as much.

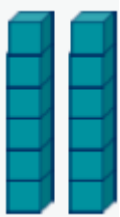
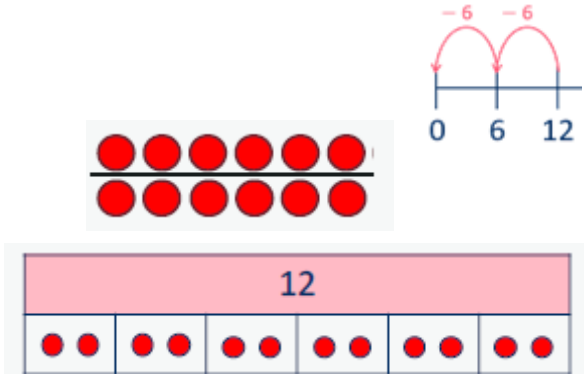



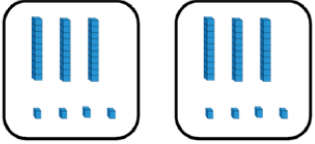
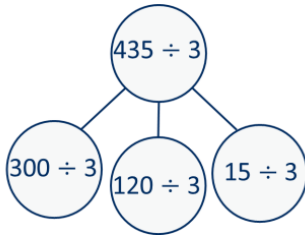
Correspondence problems
Encourage children to use tables to show all the different possible combinations.

For every ... , there are ... possibilities.
There are ... × ... possibilities altogether.

	Deep pan	Italian	Thin
Cheese	C DP	C I	C Th
Mushroom	M DP	M I	M Th
Vegetable	V DP	V I	V Th
Chicken	C DP	C I	C Th
Tuna	T DP	T I	T Th

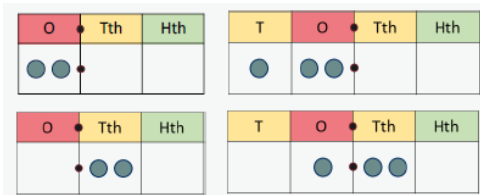
A pizza company offers a choice of 5 toppings and 3 bases.
 $5 \times 3 = 15$

Year 4 Division

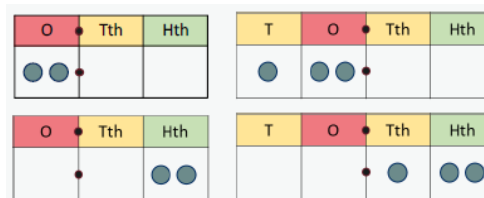
Concept	Concrete	Pictorial	Abstract
<p>Division facts to 12×12</p>	<p>There are ... groups of ... in ... $\dots \div \dots =$... has been shared equally into ... equal groups. $\dots \div \dots =$</p> 		<p>$2 \times 6 = 12$ $12 \div 6 = 2$</p>
<p>Divide a number by 1 and itself.</p>	<p>When I divide a number by 1, the number remains the same.</p> 	<p>5 shared between 1 is 5 </p> <p>There are 5 groups of 1 in 5</p> 	<p>When I divide a number by itself, the answer is 1 5 shared between 5 is 1</p>
<p>Divide 2 and 3 digit numbers using formal written method of short division with and without remainders</p>	 <p>$68 \div 2 = 34$</p>		<p>$300 \div 3 = 100$ $120 \div 3 = 40$ $15 \div 3 = 5$ $435 \div 3 = 145$</p>

Divide by 10 and 100
Encourage children to notice that dividing by 100 is the same as dividing by 10 twice.

When I divide by 10, the digits move 1 place value column to the right.
... is one-tenth the size of ...



When I divide by 100, the digits move 2 place value columns to the right.
... is one-hundredth the size of ...



$$2 \div 10 = 0.2$$

$$12 \div 10 = 1.2$$

$$2 \div 100 = 0.02$$

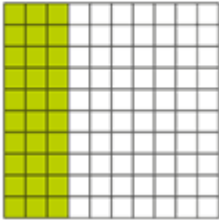



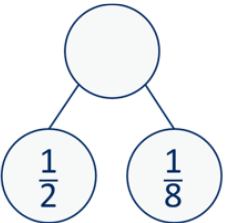
$$12 \div 100 = 0.12$$

UPPER KEY STAGE 2

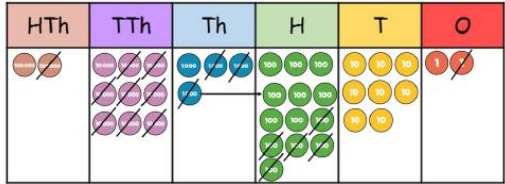
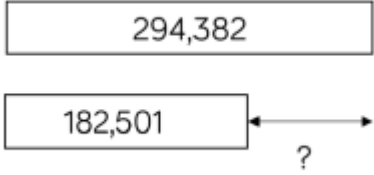
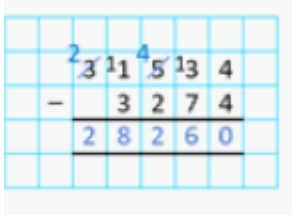
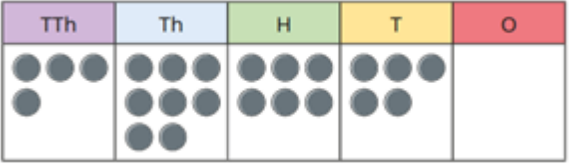
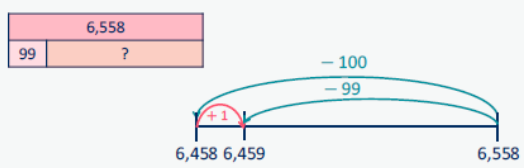
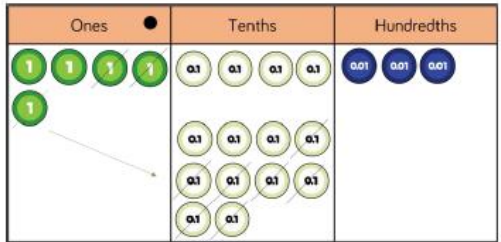
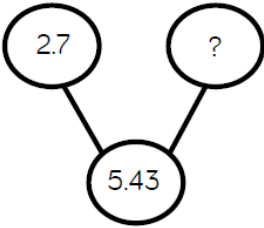
In upper Key Stage 2, children build on secure foundations in calculation, and develop fluency, accuracy and flexibility in their approach to the four operations. They work with whole numbers and adapt their skills to work with decimals, and they continue to develop their ability to select appropriate, accurate and efficient operations.

Year 5 Addition

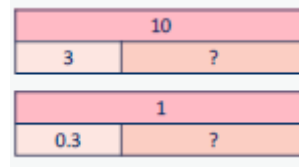
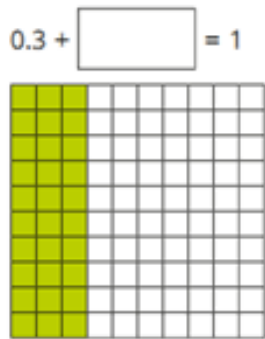
Concept	Concrete	Pictorial	Abstract																																																																					
<p>Add using mental strategies</p> <p>Add 1s, 10s, 100s, etc. to any number.</p> <p>Use number bonds and related facts.</p>			$48,650 + 300 =$ $48,650 + 30,000 =$ $48,650 + 30 =$																																																																					
<p>Add whole numbers with more than 4 digits using formal written methods</p>			<table style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>TTh</td><td>Th</td><td>H</td><td>T</td><td>O</td></tr> <tr><td></td><td>2</td><td>3</td><td>4</td><td>0</td><td>5</td></tr> <tr><td>+</td><td></td><td>7</td><td>8</td><td>9</td><td>2</td></tr> <tr><td colspan="6"><hr/></td></tr> <tr><td></td><td>3</td><td>1</td><td>2</td><td>9</td><td>7</td></tr> <tr><td colspan="6"><hr/></td></tr> <tr><td></td><td></td><td> </td><td> </td><td></td><td></td></tr> </table>		TTh	Th	H	T	O		2	3	4	0	5	+		7	8	9	2	<hr/>							3	1	2	9	7	<hr/>																																						
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<p>Add numbers with up to 2 decimal places using formal written methods</p>	<p>. I do/do not need to make an exchange because ...</p> <p>I can exchange 10 ... for 1 ...</p>	<table style="margin-left: auto; margin-right: auto;"> <tr><td>O</td><td>•</td><td>Tth</td><td>Hth</td></tr> <tr><td></td><td>•</td><td>0</td><td>9</td><td>2</td></tr> <tr><td>+</td><td>•</td><td>0</td><td>3</td><td>3</td></tr> <tr><td colspan="5"><hr/></td></tr> <tr><td></td><td>•</td><td>1</td><td>2</td><td>5</td></tr> <tr><td colspan="5"><hr/></td></tr> <tr><td></td><td></td><td> </td><td></td><td></td></tr> </table>	O	•	Tth	Hth		•	0	9	2	+	•	0	3	3	<hr/>						•	1	2	5	<hr/>										<table style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>O</td><td>•</td><td>Tth</td><td>Hth</td></tr> <tr><td></td><td>0</td><td>•</td><td>9</td><td>2</td></tr> <tr><td>+</td><td>0</td><td>•</td><td>3</td><td>3</td></tr> <tr><td colspan="5"><hr/></td></tr> <tr><td></td><td>1</td><td>•</td><td>2</td><td>5</td></tr> <tr><td colspan="5"><hr/></td></tr> <tr><td></td><td></td><td></td><td> </td><td></td></tr> </table>		O	•	Tth	Hth		0	•	9	2	+	0	•	3	3	<hr/>						1	•	2	5	<hr/>									
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<p>Complements to 1</p> <p>Pairs of numbers with up to 3 decimal places which total 1</p> <p>Encourage children to make links with bonds to 10 and complements to 100 and 1,000</p>	<p>$0.3 + \square = 1$</p> 	 	<p>$4 + 6 = 10$ $0.4 + 0.6 = 1$ $44 + 56 = 100$ $0.44 + 0.56 = 1$ $444 + 556 = 1,000$ $0.444 + 0.556 = 1$</p>
<p>Add fractions with denominators that are a multiple of one another</p> <p>Encourage children to convert fractions to the same denominator before adding.</p> <p>Progress from adding fractions within 1 whole to adding fractions beyond 1 whole.</p>	<p>The denominator has been multiplied by ... , so the numerator needs to be multiplied by... for the fractions to be equivalent.</p>  <p>$\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$</p>		<p>$\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$</p>

Year 5 Subtraction

Concept	Concrete	Pictorial	Abstract
Subtract whole numbers with more than 4 digits using formal written methods			
Subtract using mental strategies Subtract 1s, 10s, 100s etc from any number. Use number bonds and related facts.	<p>To subtract ..., I can subtract ... then add ...</p> 		$48,650 - 300 =$ $48,650 - 30,000 =$ $48,650 - 30 =$
Subtract numbers with up to 2 decimal places			$\begin{array}{r} 4 \ 1 \\ 5.43 \\ - 2.7 \\ \hline 2.73 \end{array}$

Complements to 1
Encourage children to make links with bonds to 10 and complements to 100 and 1,000 when finding a missing part or subtracting from 1

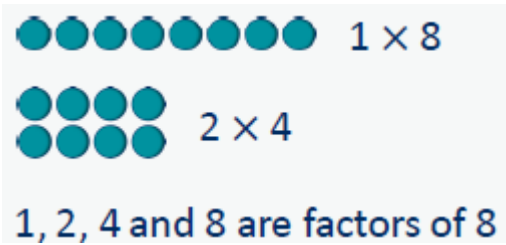


$10 - 4 = 6$ $1 - 0.4 = 0.6$
 $100 - 44 = 56$ $1 - 0.44 = 0.56$
 $1,000 - 444 = 556$ $1 - 0.444 = 0.556$

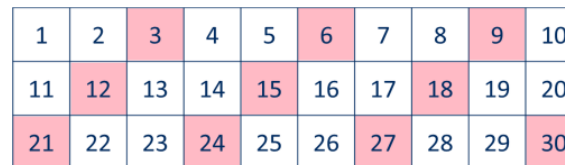
Year 5 Multiplication

Concept
Multiples and factors
Encourage children to notice patterns and make links with known facts.

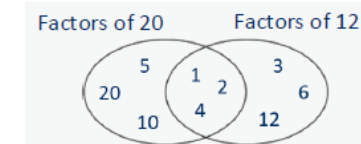
Concrete



Pictorial



Abstract

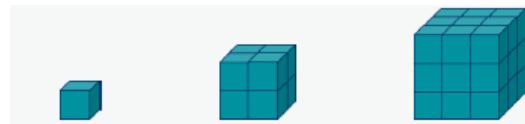


Square and cube numbers

... squared means ... x ...



... cubed means ... x ... x ...

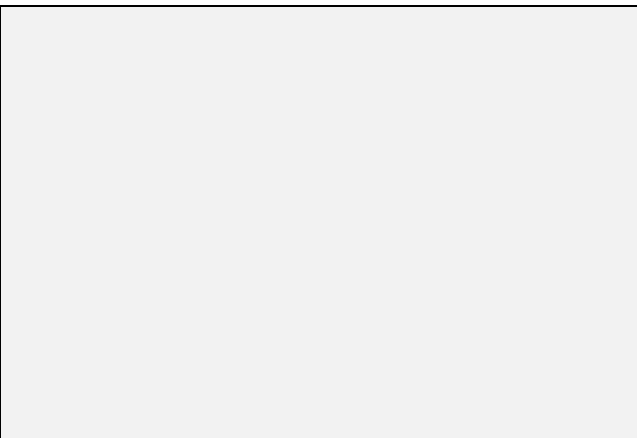


1×1 2×2 3×3
 $1^2 = 1$ $2^2 = 4$ $3^2 = 9$

$1 \times 1 \times 1$ $2 \times 2 \times 2$ $3 \times 3 \times 3$
 $1^3 = 1$ $2^3 = 8$ $3^3 = 27$

Multiply numbers up to 4 digits by a 1-digit number
This builds on the short multiplication method introduced in Y4

To multiply a 4-digit number by ... , I multiply the ones by ... , the tens by ... , the hundreds by ... and the thousands by



Multiply numbers with up to 4 digits by a 2 digit number
Numbers are first partitioned using an area model then long multiplication is introduced for the first time.

I can partition ... into ... and ...

x	10	3
30	300	90
2	20	6

(32 x 3)
(32 x 10)

300 + 90 + 20 + 6 = 416

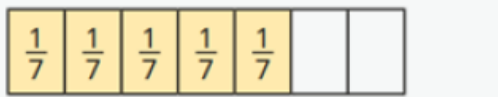
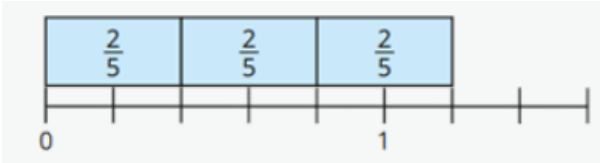
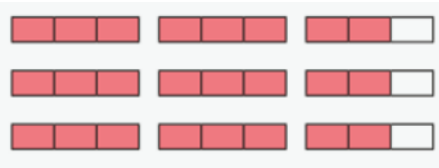
Multiply by 10, 100 and 1,000
Some children may overgeneralise that multiplying by a power of 10 always results in adding zeros. This

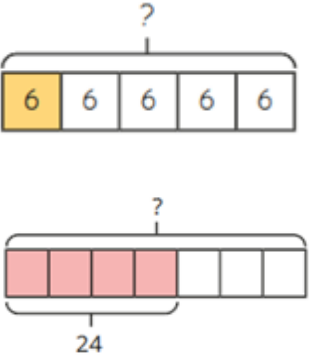
To multiply by 10/100/1,000, I move all the digits ... places to the left.
... is 10/100/1,000 times the size of ...

M	HTh	TTh	Th	H	T	O
				● ●	● ● ● ●	● ● ● ●

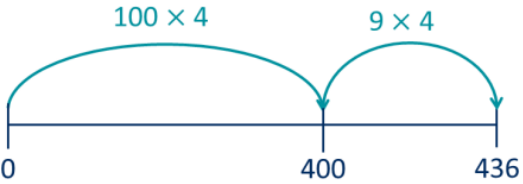
$234 \times 10 = 2,340$
 $234 \times 100 = 23,400$
 $234 \times 1,000 = 234,000$

$2.34 \times 10 = 23.4$
 $2.34 \times 100 = 234$
 $2.34 \times 1,000 = 2,340$

will cause issues later when multiplying decimals.			
Mental strategies Children continue to use efficient mental strategies such as partitioning and knowledge of factor pairs and related facts to multiply.			<p>The most efficient strategy to calculate ... × ... is ...</p> <p>To calculate ... × 12, I can do ... × ... × ...</p> <p>For example: 121 × 12 I could calculate 100 × 12 plus 20 × 12 plus 1 × 12</p> <p>I could calculate 121 × 10 plus 121 × 2 I could calculate 121 × 6 × 2 I could calculate 121 × 4 × 3</p>
Multiply fractions by a whole number Make links with repeated addition.	<p>To multiply a fraction by an integer, I multiply the numerator by the integer and the denominator remains the same.</p>  $\frac{1}{7} \times 5 = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{5}{7}$		$\frac{1}{5} \times 6 = \frac{6}{5} = 1\frac{1}{5}$
Multiply mixed numbers by a whole number			$2\frac{2}{3} \times 3$ $2 \times 3 = 6 \quad \frac{2}{3} \times 3 = \frac{6}{3} = 2$ $2\frac{2}{3} \times 3 = 6 + 2 = 8$

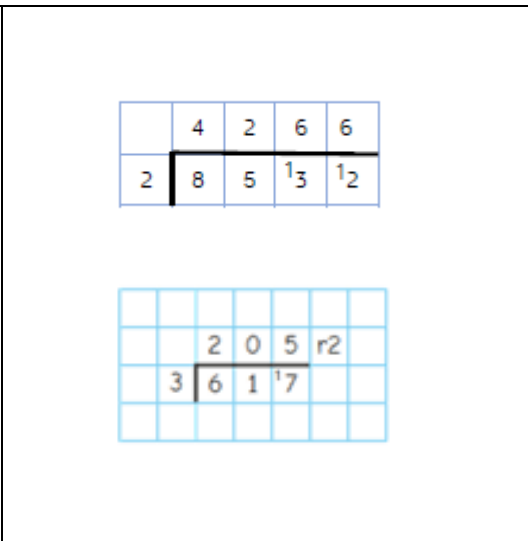
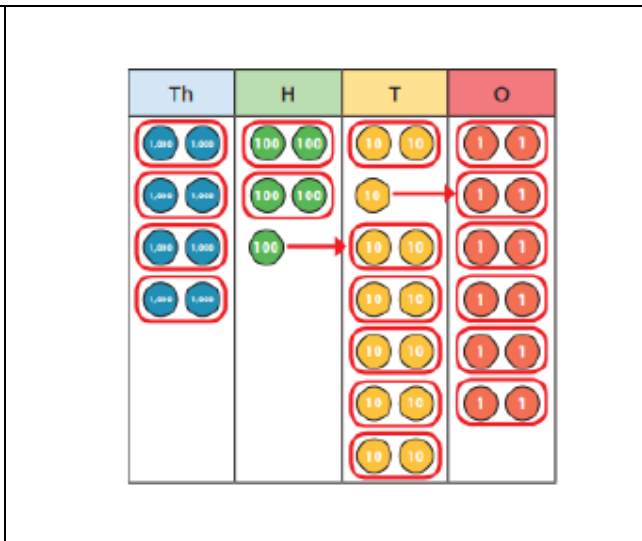
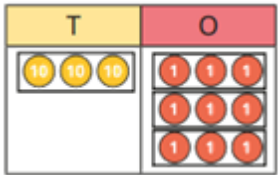
<p>Find the whole Children multiply to find the whole from a given part.</p>			<p>$\frac{1}{5}$ of ___ = 6</p> <p>$5 \times 6 = 30$</p> <p>$\frac{1}{5}$ of 30 = 6</p> <p>$\frac{4}{7}$ of ___ = 24</p> <p>$\frac{1}{7} = 24 \div 4 = 6$</p> <p>$7 \times 6 = 42$</p> <p>$\frac{4}{7}$ of 42 = 24</p>
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Year 5 Division

Concept	Concrete	Pictorial	Abstract
<p>Mental strategies</p>	<p>I can partition ... into ... and ... to help me to divide more easily</p>	<p>I can show groups of ... on a number line.</p> 	<p>To divide by ..., I can divide by ... and then divide the result by ...</p> <p>$436 \div 4 = 436 \div 2 \div 2$</p> <p>$436 \div 2 = 218$</p> <p>$218 \div 2 = 109$</p>

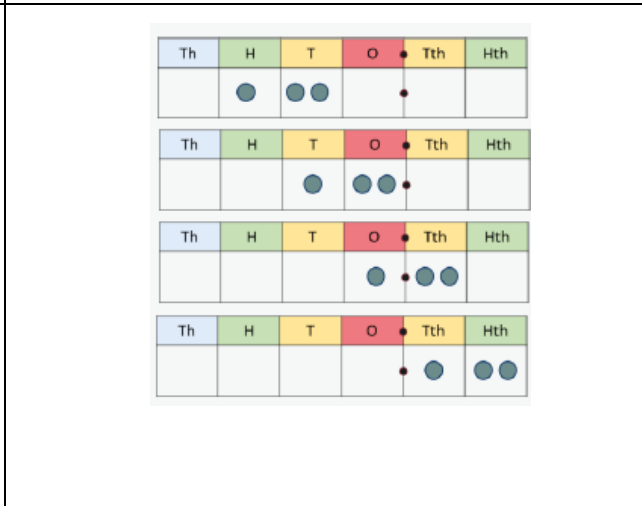
Divide up to 4 digit numbers by a 1 digit number
Short division is used for the first time

There are ... groups of ... hundreds/tens/ones/ in ...
I can exchange 1 ... for 10 ...

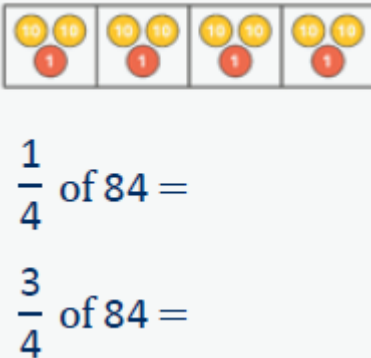
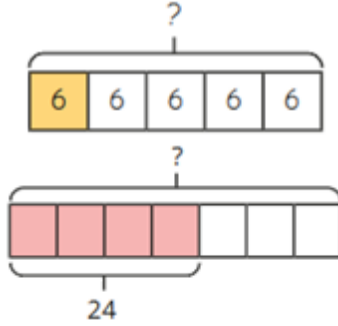


Divide by 10, 100 and 1,000
Encourage children to notice that dividing by 100 is the same as dividing by 10 twice, and that dividing by 1,000 is the same as dividing by 10 three times.

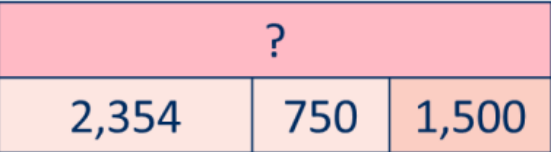
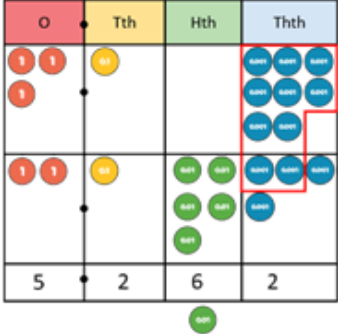
To divide by 10/100/1,000, I move all the digits ... places to the right.
... is one-tenth/one-hundredth/one-thousandth the size of

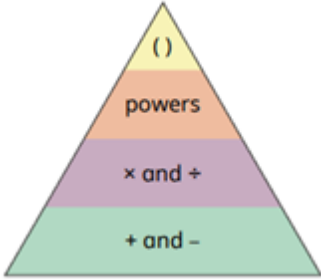

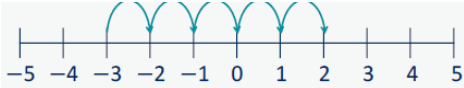
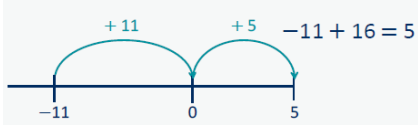



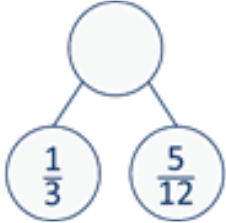
$120 \div 10 = 12$
 $120 \div 100 = 1.2$
 $120 \div 1,000 = 0.12$

<p>Fraction of an amount</p> <p>Bar models support children to understand that to find a fraction of an amount, we divide by the denominator and multiply by the numerator.</p>	 <p>$\frac{1}{4}$ of 84 =</p> <p>$\frac{3}{4}$ of 84 =</p>		<p>$\frac{1}{5}$ of ___ = 6</p> <p>$\frac{4}{7}$ of ___ = 24</p>
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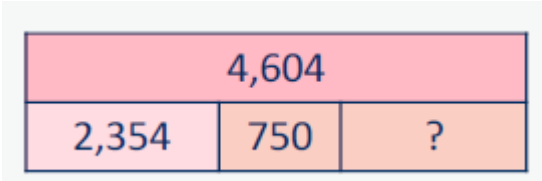
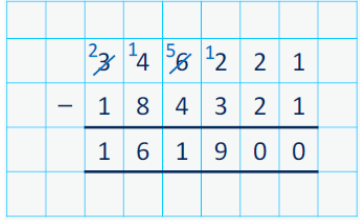
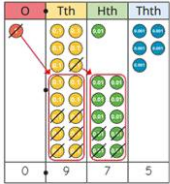
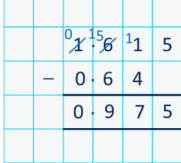
Year 6 Addition

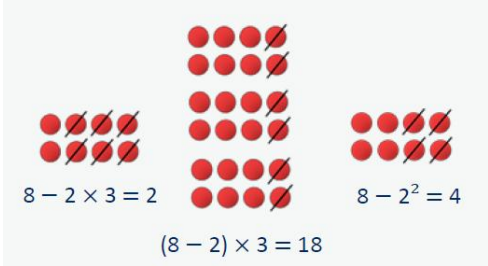

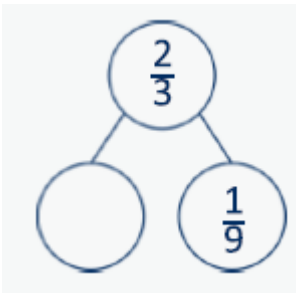
Concept	Concrete	Pictorial	Abstract																																
<p>Add integers up to 10 million</p> <p>Encourage children to estimate and use inverse operations to check answers to calculations.</p>			<table border="1" style="border-collapse: collapse; text-align: center; width: 100%;"> <tr><td></td><td></td><td>3</td><td>4</td><td>6</td><td>2</td><td>2</td><td>1</td></tr> <tr><td></td><td>+</td><td>1</td><td>8</td><td>4</td><td>3</td><td>2</td><td>1</td></tr> <tr style="border-top: 1px solid black;"><td></td><td></td><td>5</td><td>3</td><td>0</td><td>5</td><td>4</td><td>2</td></tr> <tr style="border-top: 1px solid black;"><td></td><td></td><td>1</td><td>1</td><td></td><td></td><td></td><td></td></tr> </table>			3	4	6	2	2	1		+	1	8	4	3	2	1			5	3	0	5	4	2			1	1				
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<p>Add decimals with up to 3 decimal places</p> <p>Progress to numbers with digits in different place value columns.</p> <p>Encourage children to check</p>	<p>I do/do not need to make an exchange because ...</p>		<table border="1" style="border-collapse: collapse; text-align: center; width: 100%;"> <tr><td></td><td></td><td>3</td><td>1</td><td>0</td><td>8</td></tr> <tr><td></td><td>+</td><td>2</td><td>1</td><td>5</td><td>4</td></tr> <tr style="border-top: 1px solid black;"><td></td><td></td><td>5</td><td>2</td><td>6</td><td>2</td></tr> <tr style="border-top: 1px solid black;"><td></td><td></td><td></td><td></td><td>1</td><td></td></tr> </table>			3	1	0	8		+	2	1	5	4			5	2	6	2					1									
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<p>that they have lined up the columns correctly.</p>			
<p>Order of operations Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction. *When no brackets are shown and the operations have the same priority, work left to right.</p>	<p>... has greater priority than ..., so the first part of the calculation I need to do is ...</p> 	<p>$(3 + 4) \times 2 = 14$</p> 	<p>$(3 + 4) \times 2 = 14$</p>
<p>Negative numbers Children add to negative numbers and carry out calculations which cross 0</p>	<p>... plus ... is equal to ...</p> 		<p>$-11 + 16 = 5$</p>

<p>Add fractions Convert fractions to the same denominator before adding. Progress from fractions where one denominator is a multiple of the other, to any fractions and then to mixed numbers.</p>			$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$
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Year 6 Subtraction

Concept	Concrete	Pictorial	Abstract
<p>Subtract integers up to 10 million Encourage children to estimate and use inverse operations to check answers to calculations.</p>			
<p>Subtract decimals with up to 3 decimal places Progress from the same</p>	<p>I do/do not need to make an exchange because ...</p>		


<p>number of decimal and whole number places to a different number of decimal and whole number places.</p>			
<p>Order of operations Children learn the order of priority for operations in a calculation. Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.</p>	<p>... has greater priority than ... , so the first part of the calculation I need to do is ...</p>		$8 - 2 \times 3 = 2$ $(8 - 2) \times 3 = 18$ $8 - 2^2 = 4$
<p>Subtract fractions Convert fractions to the same denominator before subtracting. Progress from fractions where one</p>			$\frac{2}{3} - \frac{1}{9} = \frac{6}{9} - \frac{1}{9} = \frac{5}{9}$

denominator is a multiple of the other, to any fractions and then subtracting from a mixed number.			
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Year 6 Multiplication

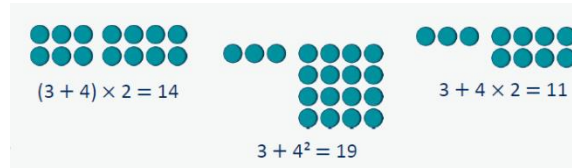
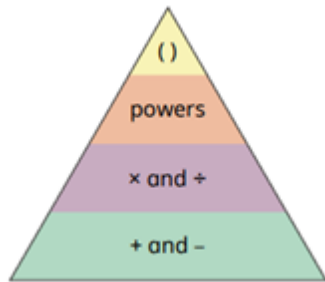
Concept	Concrete	Pictorial	Abstract
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<p>Multiply multi-digit numbers up to 4 digits by a 2 digit whole number using the formal written method of long multiplication.</p>			<p>To multiply by a 2-digit number, first multiply by the ones, then multiply by the tens and then find the total.</p> <table border="1" style="margin: 10px auto; text-align: center;"> <tr><td>TTh</td><td>Th</td><td>H</td><td>T</td><td>O</td></tr> <tr><td></td><td>2</td><td>7</td><td>3</td><td>9</td></tr> <tr><td>×</td><td></td><td></td><td>2</td><td>8</td></tr> <tr><td colspan="5"><hr/></td></tr> <tr><td>2</td><td>1</td><td>9</td><td>1</td><td>2</td></tr> <tr><td>2</td><td>5</td><td>3</td><td>7</td><td></td></tr> <tr><td colspan="5"><hr/></td></tr> <tr><td>5</td><td>4</td><td>7</td><td>8</td><td>0</td></tr> <tr><td>1</td><td></td><td>1</td><td></td><td></td></tr> <tr><td colspan="5"><hr/></td></tr> <tr><td>7</td><td>6</td><td>6</td><td>9</td><td>2</td></tr> <tr><td colspan="5"><hr/></td></tr> <tr><td colspan="5">1</td></tr> </table>	TTh	Th	H	T	O		2	7	3	9	×			2	8	<hr/>					2	1	9	1	2	2	5	3	7		<hr/>					5	4	7	8	0	1		1			<hr/>					7	6	6	9	2	<hr/>					1				
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<p>Multiply by 10, 100 and 1,000 Some children may over-generalise that multiplying by a power of 10 always results in adding zeros.</p>		<p>To multiply by 10/100/1,000, I move all the digits ... places to the left. ... is 10/100/1,000 times the size of ...</p> 	<p> $234 \times 10 = 2,340$ $234 \times 100 = 23,400$ $234 \times 1,000 = 234,000$ </p> <p> $0.234 \times 10 = 2.34$ $0.234 \times 100 = 23.4$ $0.234 \times 1,000 = 234$ </p>
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Order of operations
Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.

... has greater priority than ..., so the first part of the calculation I need to do is ...

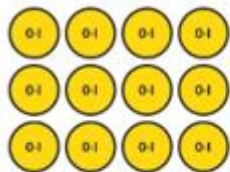


$$(3 + 4) \times 2 = 14$$

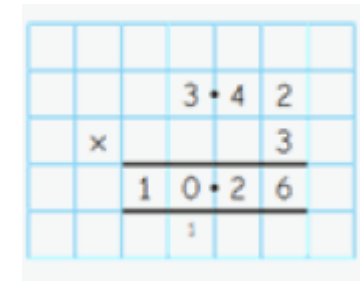
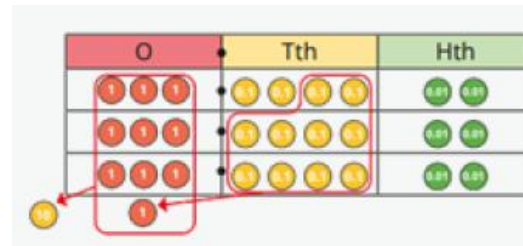
$$3 + 4^2 = 19$$

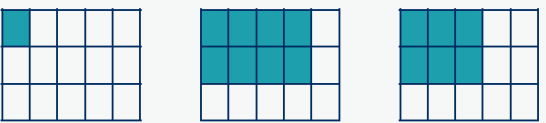
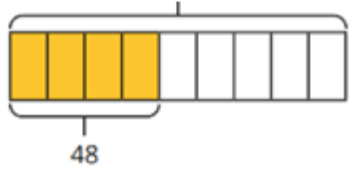
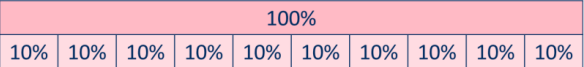
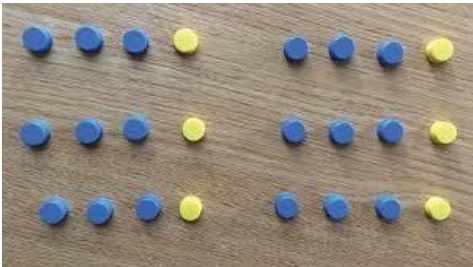


$$3 + 4 \times 2 = 11$$

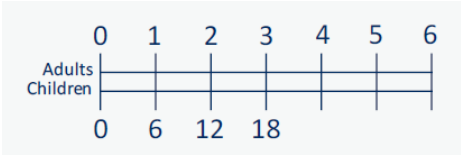
Multiply decimals by integers
This is the first time children multiply decimals by numbers other than 10, 100 or 1,000
Encourage them to make links with known facts and whole number multiplication.



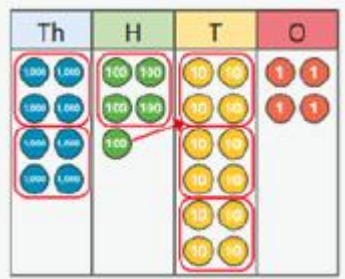
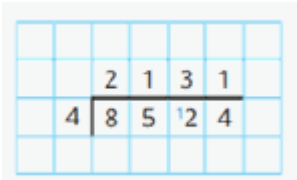
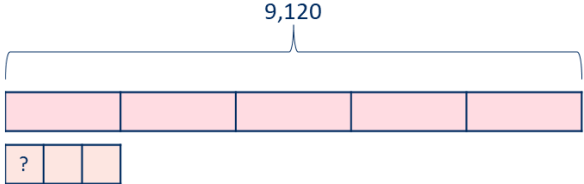
3 groups of 4 tenths is 12 tenths.
4 groups of 3 tenths is 12 tenths.

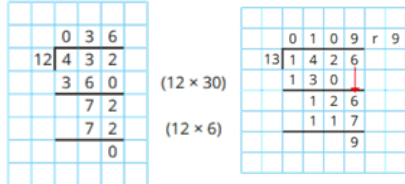
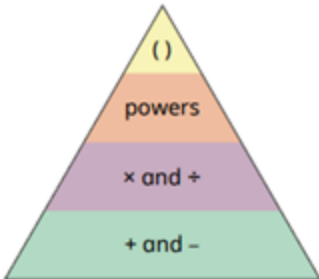
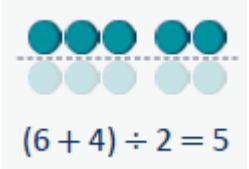



<p>Multiply fractions by fractions Encourage children to give answers in their simplest form.</p>		 <p>$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$ $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$ $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$</p>	$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$
<p>Find the whole Children multiply to find the whole from a given part.</p>			$\frac{1}{9} = 48 \div 4 = 12$ $9 \times 12 = 108$ $\frac{4}{9} \text{ of } 108 = 48$
<p>Calculate percentages Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.</p>			<p>To find 30%, I can find 10% and then multiply it by 3 To find 23%, I can use $10\% \times 2$ and $1\% \times 3$ To find 99%, I can find 1%, then subtract from 100%</p> $50\% \text{ of } \dots = \dots \div 2$ $25\% \text{ of } \dots = \dots \div 4$
<p>Calculations involving ratio Encourage children to see the multiplicative relationship between ratios. They will need to multiply</p>	<p>For every ... , there are ...</p> 	<p>For every 1 adult on a school trip, there are 6 children.</p> <p>adults </p> <p>children </p>	<p>The ratio of adults to children is 1 : 6</p>

<p>or divide each value by the same number to keep the ratio equivalent. Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.</p>			
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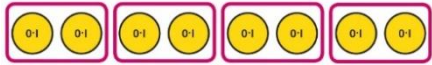
Year 6 Division

Concept	Concrete	Pictorial	Abstract
<p>Short division Encourage children to interpret remainders in context.</p>	<p>There are ... groups of ... hundreds/tens/ones/ in ... I can exchange 1 ... for 10 ...</p>		
<p>Mental strategies Include partitioning and number line strategies outlined in Y5 as well as</p>	<p>To divide by ... , I can first divide by ... and then divide the answer by ...</p>		<p>$9,120 \div 15 = 9,120 \div 5 \div 3$</p>

<p>division using factors.</p>			
<p>Long division The long division method is introduced for the first time. Two alternative methods are shown.</p>			
<p>Order of operations Calculations in brackets should be done first, then powers. Multiplication and division should be performed before addition and subtraction.</p>	<p>... has greater priority than ..., so the first part of the calculation I need to do is ...</p> 		<p>$(6 + 4) \div 2 = 5$</p>
<p>Divide by 10, 100 and 1,000 Encourage children to notice that dividing by 100 is the same as dividing by 10</p>	<p>To divide by ..., I move the digits ... places to the right.</p>		<p>$312 \div 10 = 31.2$ $312 \div 100 = 3.12$ $312 \div 1,000 = 0.312$</p>

twice, and that dividing by 1,000 is the same as dividing by 10 three times.

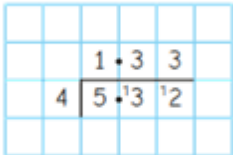
Divide decimals by integers
This is the first time children divide decimals by numbers other than 10, 100 or 1,000



Use place value equipment to divide decimals into equal groups.
8 tenths divided into 4 groups. 2 tenths in each group.

0.8			
?	?	?	?

$4 \times 2 = 8$ $8 \div 4 = 2$
 So, $4 \times 0.2 = 0.8$ $0.8 \div 4 = 0.2$



Decimal and fraction equivalents

The fraction ... is equivalent to the decimal ...

1					
$\frac{1}{2}$			$\frac{1}{2}$		
$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$	
$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$	
$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$	


1							
0.5				0.5			
0.25		0.25		0.25		0.25	
0.2		0.2		0.2		0.2	
0.1		0.1		0.1		0.1	

$\times 25$
 $\frac{3}{4} = \frac{75}{100} = 0.75$
 $\times 25$


$\frac{1}{5} = 0.2$ $\frac{2}{5} = 0.4$ $\frac{3}{5} = 0.6$

Divide a fraction by an integer
This is the first time children divide fractions by an integer.

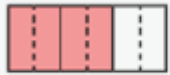
... ones divided by 2 is ... ones
so ... sevenths divided by 2 is ... sevenths.



I am dividing by ... , so I can split each part into ... equal parts.

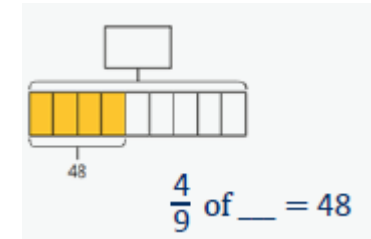
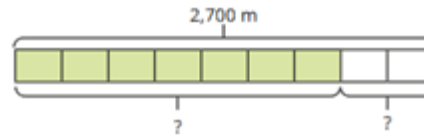


... is equivalent to ...
so ... \div ... = ... \div ...



$\frac{2}{3} = \frac{4}{6}$
 so $\frac{2}{3} \div 4 = \frac{4}{6} \div 4 = \frac{1}{6}$

Fraction of an amount
Children divide and multiply to find fractions of an amount. Bar models can still be used to support understanding where needed.

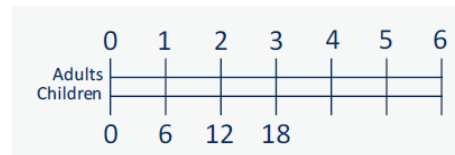
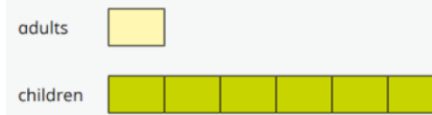


Calculations involving ratio
Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent. Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.

For every ... , there are ...



For every 1 adult on a school trip, there are 6 children.



The ratio of adults to children is 1 : 6

Calculate percentages
 Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.

There are ... lots of ... % in 100%
 To find ... %, I need to divide by ...

100%			
50%		50%	
25%	25%	25%	25%

100%									
10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

To find 30%, I can find 10% and then multiply it by 3
 To find 23%, I can use $10\% \times 2$ and $1\% \times 3$
 To find 99%, I can find 1%, then subtract from 100%

50% of ... = $\dots \div 2$
 25 % of ... = $\dots \div 4$